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Some Notes and Equations for Forward Scatter compiled by James Richardson.

Here are some basic notes on the canonical equations for meteor forward-scatter which I originally put together for another email list, but which I thought might be of interest here as well. There is a little math involved, but the information which can be gathered from the equations is quite informative as to how a forward scatter system will behave under different system and link configurations (on the ground), and different meteor velocities and flight directions (in the atmosphere).
The basic geometry requirement for forward-scatter is as follows: In order to cause a forward scatter reflection, the meteor trail must lie within a plane (called the tangent plane) which is tangent to an ellipsoid having the transmitter and receiver as its foci. The entire reflection path will also lie within a plane (called the plane of propagation), which contains the transmitter, reflection point, and receiver.
The plane of propagation will be normal to (at right angles to) the meteor tangent plane. Important note: the meteor itself can be at any orientation within the tangent plane -- it need not be normal itself to the propagation path.
There is, however, greater signal loss when the meteor trail is perpendicular to the propagation plane than when it is parallel to the propagation plane.
A third useful constraint is that most meteor reflections will Occur within the narrow altitude band of about 85 to 105 km altitude.
Thus, the sphere formed by the 95 km altitude band, the meteor tangent plane, and the ellipsoid having the transmitter and receiver as foci must all meet (or be tangential) at the reflection point. Another often quoted set of thumb rules for radiometeor reflections are the proportionalities concerning the used radio frequency wavelength and echo power, duration, and echo numbers.
These are:

- •The echo power is proportional to lambda^3

- •The echo duration is proportional to lambda^2

- •The number of echoes is roughly proportional to lambda where:

lambda = transmitted RF wavelength

But these thumb rules only tell a portion of the story, and it is necesary to dig in a little deeper to gain a working understanding of how to optimize a particular link setup. For this presentation, I draw heavily upon the radiometeor enthusiast's "Bible" -- "Meteor Science and Engineering," D.W.R. McKinley, (McGraw-Hill, 1961).
These notes come from Chapter 8 (on back-scatter) and Chapter 9 (forward-scatter), and those who have access to this book are strongly encouraged to verify my notes and inspect the accompanying figures.
The "classical" equations for forward-scatter from a meteor trail, which have been derived from theory and validated empirically during the heyday of radiometeor astronomy (1945- 1970) , are as follows:

- Underdense trails (electron line density, Q < 1E14 electrons / meter)

Underdense Echo Power
The echo power received at the receiving station in a forward Scatter underdense echo is given by (Eq. 9-3, page 239), as the product of two fractions:
P_r = ((P_t * g_t * g_r * lambda^3 * sigma_e) / (64pi^3)) * ((Q^2 * sin^2(gamma)) / ((r1 * r2) * (r1 + r2) * (1 - sin^2(phi) * cos^2(beta)))), where: P_r = power seen by receiver (Watts), P_t = power produced by transmitter (Watts), g_t = gain of transmitting antenna, g_r = gain of receiving antenna, lambda = RF wavelength (m), sigma_e = scattering cross section of the free electron (m^2), Q = electrons per meter of path, r1 = distance between meteor trail and transmitter (m), r2 = distance between meteor trail and receiver (m), phi = angle between r1 line and normal to meteor path tangent plane, or phi = 1/2 angle between the r1 and r2 lines, beta = angle between meteor trail and the intersection line of the tangent plane and plane of propagation, gamma = angle between the electric vector of the incident wave and the line of sight to the receiver (polarization coupling factor).
A useful substitute for sigma_e is: sigma_e = 1.0E-28 * sin^2(gamma) m^2, which reduces in the back-scattter case to simply: sigma_e = 1.0E-28 m^2.

- Underdense Echo power decay

A second useful expression from this chapter for the exponential decay over time of the underdense echo power is given by (Eq. 9- 4, page 239), as an exponential (e^x) raised to a fraction): P_r(t)/P_r(0) = exp(-(((32pi^2 * D * t) + (8pi^2 * r0^2)) / (lambda^2 * sec^2(phi)))), where: P_r(t)/P_r(0) = normalized echo power as a function of time (t), t = time in seconds (sec), D = electron diffusion coefficient (m^2/sec), r0 = initial meteor trail radius (m).
The diffusion coefficient, D, will increase roughly exponentially with height in the meteor region. An empirical derivation from Greenhow & Nuefeld (1955) is given for meteor altitudes of h = 80 km to h = 100 km: log10(D) = (0.067 * h) - 5.6, for D in m^2/sec. The initial meteor trail radius is another empirically derived value, given in two studies as:
1956 & 1959 ARDC data;
log10(r0) = (0.075 * h) - 7.2, h = meteor altitude (75-120 km) r0 = trail radius (m) * Manning (1958); log10(r0) = (0.075 * h) - 7.9.
Underdense echo duration An approximate expression for the duration of an underdense trail is given by Eq. 9-6, page 240: t_uv = (lambda^2 * sec^2(phi)) / (16pi^2 * D) ** Overdense trails (electron line density, Q > 1E14 electrons / meter): The classical expressions for the overdense trails contain many More assumptions and estimations than for the underdense trails. Their full theory is still under development today. However, the classical equations can still be used to glean some of the basic characteristics of these events. I am showing these here in their final form, skipping some intermediate steps and approximations. * Overdense echo power This is Eq. 9-7 on page 242: P_r = 3.2E-11 * ((P_t * g_t * g_r * lambda^3 * Q^(1/2) * sin^2(gamma)) / ((r1*r2) * (r1+r2) * (1 -sin^2(phi) * cos^2(beta)))). * Overdense Echo Duration An approximate expression for overdense echo duration is given by Eq. 9-8 on page 242: t_ov = 7E-17 * ((Q * lambda^2 * sec^2(phi)) / D). ** General Notes A few of the more important relationships from these equations are: * Note that the thumb rules initially given concerning wavelength, lambda, are verified in these equations, at least for echo power and duration. * The electron line density, Q, is a function of the meteor mass , velocity, and composition, much as is meteor magnitude. Some important relationships from the above equations can be gleaned: -- for underdense trails; Echo power is proportional to Q^2 Echo duration is independent of Q (!) -- for overdense trails; Echo power is proportional to Q^(1/2) Echo duration is proportional to Q These correlations were used as one of the criteria for Statistically separating underdense from overdense echoes recorded at Poplar Springs, Florida. * The diffusion coefficient, D, and initial trail

radius, r0, are the primary reasons for the well known "height-ceiling" effect in forward-scatter systems. Most systems are limited to an effective ceiling of about 105-110 km above which echoes cannot normally be detected. The trail radius becomes a limiting factor due to electron density decrease and destructive interference between the reflections from different portions of the trail at the first Fresnel zone -- front to back and side to side. The diffusion coefficient, D, decreases the amount of time it takes for the trail to reach these poor reflection conditions. Additionally, there is also a "hight-floor" effect seen in slow, overdense trails, which begins to seriously decrease their durations when the trail altitudes drop to about the 80-85 km altitude level. This is also currently under investigation, and is thought to be due to the more rapid free electron recombinations and attachments at this lower altitude (higher air density) region. The upshot of these two effects is that most forward-scatter systems tend to be more sensitive to meteors which occur in the 85-105 km altitude band, with an average of about 95 km. This makes the systems most responsive to medium-speed meteors of most magnitude levels, but somewhat discriminatory against fast, faint meteors and slow, bright meteors. * An interesting relationship is that found for the meteor trail orientation with respect to the plane of radio wave propagation, Beta. The rather anti-intuitive effect is that a higher peak reflected power will occur from a trail which is parallel to the plane of propagation, with a somewhat lower power being reflected from a trail which is perpendicular to the plane of propagation (all else held constant). ** The Secant Squared Phi Effect The key ingredient which attracted early researchers to the possibilities of radiometeor forward scatter -- both in the realm of meteor science and meteor burst communication -- was the sec^2(phi) terms which appear in the duration equations for both the underdense and overdense expressions. Additionally, helpful sin^2(phi) terms also appear in the expressions for echo peak power. What this implies is that the further transmitter and receiver are from each other, The more power the meteor trail will reflect, and the *much* longer will the duration of the echo be. At some point, the attenuation due to distance (the (r1*r2)*(r1+r2) terms) will override the advantage of continuing to increase distance and phi, but for a time (depending upon transmitter power) the advantage over the back- scatter condition is significant. This can be illustrated (and is in Chapter 9) by looking at the Best regions of atmosphere to point a transmitting and receiving antenna for a particular forward-scatter link, that is, where the highest number of echoes, highest powers, and longest durations will be obtained. if the sky is uniformly filled with meteor radiants, the highest concentration of potential reflection-causing meteor trails (those which have the proper geometry) will be located in an elliptical ring at the 95 km altitude level, having transmitter and receiver as foci. This ring corresponds to radiants having angular altitudes of about 30-60 deg, peaking near 45 deg. If the forward-scatter link is short, the elliptical ring will be fairly uniform in meteor density, but if the link is long, the ring will show higher concentrations of likely echo candidates closer to the ends of the ellipse major axis -- nearer to the vicinities of the transmitter and receiver on the ground. This would tend to support the common desire among radiometeor amateurs to point their receiving antennas at some very high elevation angle in order to catch these end-point reflections. The effect of angle Beta, discussed above, would also tend to support this notion, since a higher proportion of end-point meteors will have lower Beta's. HOWEVER, when the effect of the reflection angle, phi, is taken into account, this picture shifts very abruptly. Meteor trails located near the midpoint between the two stations will have the highest phi's, and thus give back the best power levels and significantly longer echo durations. Meteors located near the path endpoints will have lower reflected powers and much shorter durations. As an example, echoes from the midpoint region of a 600 km link will have durations about 15 times longer than echoes from the endpoint regions, while echoes from the midpoint region of a 1200 km link will have echo durations which are about 92 times longer than those echoes from the endpoint regions. The effect is that the regions of best echo characteristics will be the so-called hot spot regions, located about 50-100 km to either side of the transmitter-receiver great circle path midpoint. McKinley shows some very nice theoretical echo density maps for this type of situation, and meteor burst communication firms make almost exclusive use of hot spot reflections. This is not to say that end-point reflections do not occur; I do know of one military sponsored forward scatter experiment using a hardened below-ground antenna for meteor burst communication employing endpoint reflections, but this was a rather singular effort. For most medium and long distance forward-scatter links, relatively low antenna elevation angles, with transmitting and receiving antennas aimed at one or both hot spot regions, yield the best and most consistent results. The one exception that I know of is for a very short-range link (less than about 150 km), in which better performance in the northern hemisphere is gained by pointing the transmitting and receiving antennas to the north in order to take advantage of the higher concentration of ecliptical radians to the south. This special case is more akin to the back-scatter situation, in which phi will always be quite small, and the highest concentration of echo candidates should be sought. The below table lists the elevation angles (measured from the horizon), and relative azimuths (measured from the bearing of the great circle path between receiver and transmitter) needed to point the beam of a transmitting/receiving antenna at the cen er of the hot spot region for a particular forward-scatter link. These are given for a variety of link great circle distances. This model was created in a Maple worksheet, and gives the reflection location (altitude and azimuth) for a meteor trail occurring midway between transmitter and receiver, having a radiant at 45 deg elevation, and a flight path perpendicular to the plane of propagation. Such a meteor trail is indicative of a reflection from the center of one of the two hot spot regions for the given link. The two ngles are shown in degrees. Note the rapid drop in antenna beam elevation angle.

RANGE (km)	ALTITUDE	AZIMUTH OFFSET
50	44	75
100	41	62
150	38	51
200	34	43
250	30	37
300	27	32
350	24	29
400	22	26
450	20	23
500	18	21
550	17	20
600	15	18
650	14	17
700	13	16
750	12	15
800	11	15
850	10	14
900	9	14
950	9	13
1000	8	13
1050	8	12
1100	7	12
1150	6	12
1200	6	11
1250	6	11
1300	5	11
1350	5	11
1400	4	11
1450	4	10
1500	4	10
2000	1	10